THINGS

of science

CURVES

CURVES

The geometric curves discussed in this THINGS of science unit can be drawn in a variety of ways. They can be constructed by using pencil, straightedge, thumb tacks and string. They can be made by sewing cardboard with thread. They can be formed by folding paper and straightening it out again, then noticing the figures created by the creased lines.

The curves which you will construct are called conic sections. So that you can construct curves by paper-folding and by curve-stitching, we have included the following items in this unit:

E M B R O I D E R Y THREAD—Two skeins.

SEVEN DIAGRAMS—
 ELLIPSE
 PARABOLA
 HYPERBOLA
 SUPPORTS—Two
 CIRCLES—Two circles on one card

In addition you will need a pencil, a straightedge, a pair of scissors, several pieces of cardboard, needle, wax paper, tissue paper and thumb tacks.

The printed lines for the curves, ellipse, parabola and hyperbola, do not resemble the curves named, but as you stitch with

needle and thread, the desired conic sections will take shape (Fig. 1).

ELLIPSE

PARABOLA

HYPERBOLA
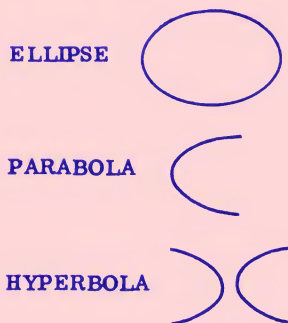
Fig. 1

The other diagrams are to be used in making a three-dimensional complete (double) cone. This will help you see how these conic sections are related to each other.

PARABOLA

Experiment 1. First make a parabola by curve-stitching. Use a long double strand length from your thread and tie a big knot at the end. On your piece of cardboard marked "Parabola," stitch from 1 to 1, 2 to 2 and so on. When you have

finished stitching, your stitches should outline a parabola, or to be more exact, should form the envelope of a parabola. The two sides of a parabola if extended will never meet.

Experiment 2. If you want to curve-stitch a larger parabola, you can easily draw your own guide lines and locate equally spaced points by following the diagram on your card.

Experiment 3. Form a parabola by paper-folding. Wax paper should be used for the paper-folding experiments because a crease shows up so clearly on it. Cut a piece of wax paper into several uncreased rectangles about 4 x 5 inches in size. Midway between two sides and about 1¼-inch from the longer edge, mark a point F (Fig. 2).
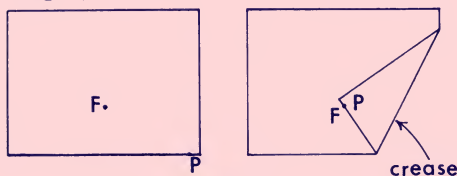
Fig. 2

Fold the paper so the dot touches the longer edge of the wax paper near one end and crease the fold. Move the dot along

the edge about ¼-inch and crease the new fold. Slide the dot progressively along the edge creasing the folds formed at ¼-inch intervals until the dot has traveled all along the edge. As you work, your creased lines begin to outline a parabola.

Experiment 4. Make several other parabolas by paper-folding by placing point F closer to the edge and further away from the edge. What happens to the curve?

ELLIPSE

Experiment 5. To curve-stitch an ellipse, you must connect many points on a circle. The lines for locating these points have already been drawn on your card marked "Ellipse." They were made by drawing through F, which can be any point inside the circle, a line intersecting the circle at two points. Find on your diagram points 1 and 1' located in this manner and circle these points. From 1 and 1', lines were drawn through O, the center of the circle. These lines are diameters of the circle. The new intersections of these lines with the circle are labeled 1' and 1 respectively. These give you two points close together; each labeled 1, and two points on the opposite side of the circle each labeled 1'. Another line

through F locates points 2 and 2′.

Stitch from 1 to 1, 1′ to 1″, 2 to 2, etc., until all the pairs of numbers are connected. Your stitches should form an ellipse with one focus at F. The other focus is at a corresponding point on the other side of O. Notice that the ellipse is open near points A and B. You can close the ellipse further by locating additional pairs of points near A and B, but to outline the ellipse completely would require an infinite number of pairs of points.

Experiment 6. Curve-stitch a more elaborate ellipse by locating pairs of points as in Experiment 5. If you do not have a compass to make your circle, place a loop of thread around a thumb tack in the center of a piece of cardboard, stick a pencil in the loop and keep the thread taut as you move the pencil over the paper.

F can be any point inside your new circle. The further F is from O, the more elongated your ellipse will be. Curve-stitch several ellipses to show that this is true.

Experiment 7. To make an ellipse by paper-folding, draw a large circle on one of the 4 x 5-inch rectangles of wax paper. Designate the center O. Mark a point F inside your wax paper circle. Fold the paper so that F touches some point on the

circle and crease the paper. Repeat this operation for many points around the entire circle, at intervals of about ¼-inch, until the ellipse becomes visible. F and O are the foci of your ellipse.

HYPERBOLA

Experiment 8. Form a hyperbola by folding paper. Draw a circle on wax paper, and this time place a point F outside the circle. Fold F over to touch many points all around the circle and crease each fold. If you keep folding and creasing, eventually both branches of the hyperbola will appear. A hyperbola always has two identical branches.

Experiment 9. To curve-stitch a hyperbola, you begin with a point F outside a circle. The lines for locating these points have already been drawn on your card marked "Hyperbola." Lines through F were drawn to intersect the circle twice. Locate on your diagram points 1 and 1′ found in this manner. From these two points were drawn diameters which intersect the circle at points 1′ and 1 respectively. In this manner many pairs of points were located.

Stitch from 1 to 1, 1′ to 1′, 2 to 2, etc., but this time extend your stitch beyond

the circle as shown. The figure outlined by the extended stitches will form a hyperbola. Does your hyperbola have two branches? The point F is one focus. The other focus will be a corresponding point F′ lying on the other side of the circle.

Experiment 10. Create several other hyperbolas by curve-stitching. Make the circles the same size each time, but change the location of the point F. How does the position of F affect the shape of the hyperbola?

THREE-DIMENSIONAL FIGURES

Your other diagrams will be used to construct three-dimensional figures to help you understand the relationship between the various conic sections you have just made. The first will be a cylinder with external supports. Figure 3 shows you how to put it together.

Experiment 11. Cut circles and supports along the solid lines. With a needle punch a hole through each dot within the circles so that you can sew through them more easily. The other two pieces of cardboard are your supports for the figure.

Score the cardboard marked "Support" along the dotted line at the center using a ruler and dull knife to make it easier to
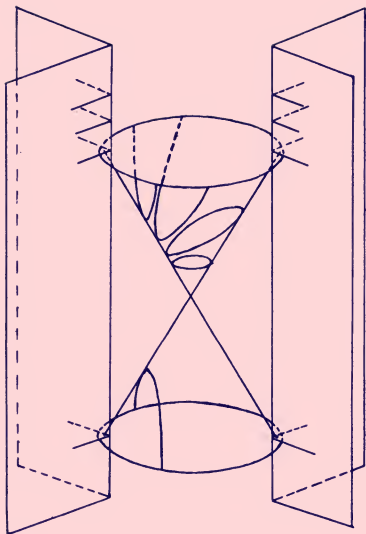
Fig. 3

bend back. Fold both supports, then cut
along the short, solid lines to make notches
in which to fit your cardboard disks. The
end with the three notches is the upper
end of the support. Insert one disk in the
top notch of each support and place the
other in the single bottom notch of each

support. Your figure should now stand up by itself.

Align the dots so those in the top disk are directly over those in the bottom disk. Sew the two circles together with a long thread, pulling the thread tight as you work. If your stitches are vertical and parallel, the surface outlined by the thread is a right circular cylinder.

Experiment 12. Remove the top cardboard disk from the notches, twist it slightly and insert it in the middle set of notches. The twisting of the cylinder gives the surface a curved appearance, but notice that each thread is quite straight. A surface that can be created by a moving straight line is called a "ruled surface." The figure that you have just formed is a hyperboloid of one sheet.

Experiment 13. Twist the top disk still further so that the strings meet at a point midway between the bases, and hold the disk in place in the lower notches. The figure now becomes a double or complete cone.

Hold a pencil in various positions behind the cone and picture the shape of the conic section if the cone were actually "cut" by a plane passing through the pencil and your eye. If you hold the pencil

so that the plane is parallel to the table or desk on which the figure stands, the section formed is a circle.

Tilt your pencil a bit and notice that if the embroidery thread were cut by a plane passing through the pencil, the figure formed would be an ellipse rather than a circle (Fig. 3).

Tilt the pencil still further until it is parallel to one of the strings, and notice that the section formed is a parabola. Tilt the pencil still more so that it cuts both parts (nappes) of the cone, and you have the familiar two-branched hyperbola (Fig. 3).

This experiment explains why the curves discussed in the unit are called conic sections.

DEFINITIONS

Experiment 14. An ellipse is relatively simple to draw with pencil, string and paper. You draw it pretty much the same way you would a circle, only loop the thread around two thumb tacks F and F' instead of just one (Fig. 4).

Experiment 15. An ellipse is usually defined as the path (called the locus in mathematics) of a point which moves in such a way that the sum of its distances
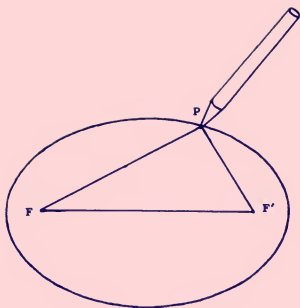
Fig. 4

from two fixed points is constant. Your thumb tacks are fixed points, or foci. The length of the loop, called L, always remains the same. When you pull the string taut with a pencil at P, the sum of the distances FP and F'P equals the length of the loop, L, minus the distance FF'. This is true for every position of the point P. Thus if you move the pencil, keeping the string taut, all points on the curve are located so that the sum of the distances from P to F and F' is a constant. Experiment and observe that this is true.

Experiment 16. To understand why it is so easy to form an ellipse by paper-

folding, fold an ellipse with a piece of thin paper such as tissue paper or onionskin paper. Draw a circle with center O (remember you began your ellipse by drawing a circle on wax paper). Mark any point F inside your circle, and select a point P on the circle. Fold P over so that it touches F and crease. Mark the points where this creased line cuts the circle A and B. Draw a line between O and P, and mark C where this line crosses the creased line. With a straight line connect F with C, and F with P. Your diagram is now complete (Fig. 5).

Now OC plus CP is the radius of your circle, and a radius is of constant length.
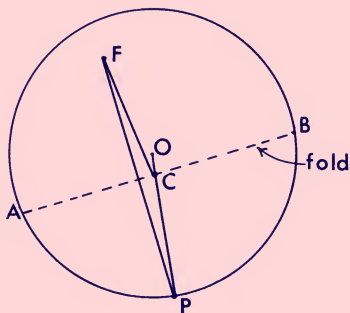

Fig. 5

Since the points P and F touched when the crease was formed, the distances from any point on AB to P and F are equal. Therefore, CP equals CF. Thus OC plus CF also equals the radius of the circle. C, by definition of the ellipse, must be a point on the ellipse with O and F as foci. There will be such a point on each crease. The figure formed by these points will be an ellipse. Continue folding the ellipse and see that this is true.

Experiment 17. A parabola is usually defined as the path or locus of a point which moves so its distances from a fixed point and a fixed line are equal. The point is called the focus, and the line the directrix.

A parabola is much easier to curve-stitch or paper-fold than it is to construct with pencil, paper and straightedge. Use a draftsman's right-angled triangle if you have one; if not, cut a right-angled triangle from a rectangular piece of cardboard, using the cardboard rectangle as the right angle. Loop one end of a piece of embroidery thread over the smallest angle of the triangle. Make a loop in the other end of the thread so it just reaches the base of the triangle and place this loop over a thumb tack stuck
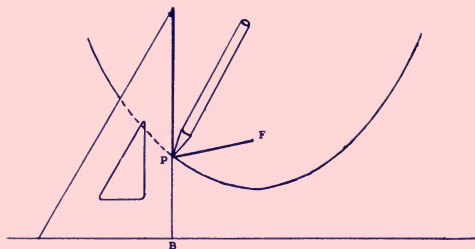
Fig. 6

into a piece of paper at F (Fig. 6). Draw
a line somewhere near F, and slide the
base of your triangle along it. If you keep
the string taut with a pencil held next to
the triangle, as you move the triangle, the
pencil will trace a parabola. Since the
length of the string does not change, the
distance of the point P from F will always
equal its distance from the line as you
move the triangle along the line.

Experiment 18. To understand why it
is so easy to form a parabola by paper-
folding, draw a straight line L and mark
a point F. Fold F so that it touches P, any
point on the line, crease the paper and
mark the line formed AB. Draw the line
PF. At P draw a perpendicular to L which
crosses AB. Mark this point C. Connect C
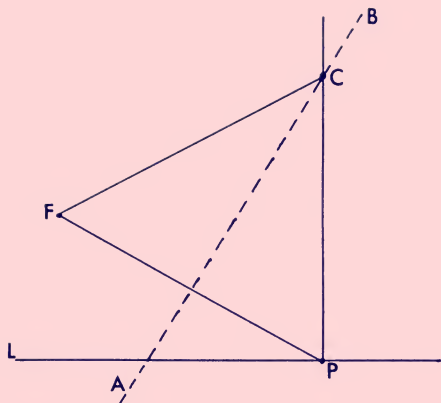
Fig. 7

with F. Then, as in the ellipse, CP equals CF. Thus the point C is equally distant from F and the line L (Fig. 7). The figure formed by all such points will be a parabola.

Experiment 19. In a hyperbola, the difference of the distances of a moving point from two fixed points is constant. From a piece of cardboard cut a straightedge about an inch wide and six inches long, but at one end cut a bulge so that F′, a point on the bulge, will be exactly in line with the rest of the straightedge (Fig.

8). At F' stick a thumb tack through the cardboard into your paper beneath so you can rotate the straightedge about the point F'. At the opposite end from F', cut a notch in your cardboard straightedge a quarter-inch or so from the end and tie embroidery thread about it, pulling it tight. Loop the string about a thumb tack at F, making the length of the string between the two loops, AC, a little shorter than the overall length of your straightedge. Keep the string taut with a pencil held against the straightedge, and as you rotate the straightedge you will draw one branch of a hyperbola. If the string is kept taut by the pencil at P, PF will equal PC.
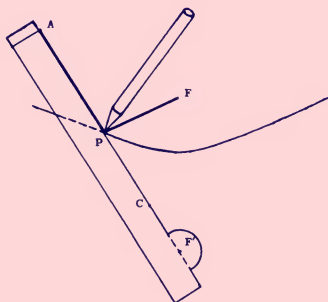
Fig. 8

Since PF′ minus PC is a constant, PF′ minus PF is also a constant. Thus P is always located so the difference of its distances from F′ and F is constant. F and F′ are the foci of your new figure. To draw the other branch of the hyperbola, pivot the straightedge at F and attach the string at F′.

Experiment 20. To demonstrate why you can paper-fold a hyperbola, again draw a circle and mark a point F outside it. Fold F so it touches P, any point on the circle, and crease the line AB. Draw PO, a radius of the circle, and extend be-
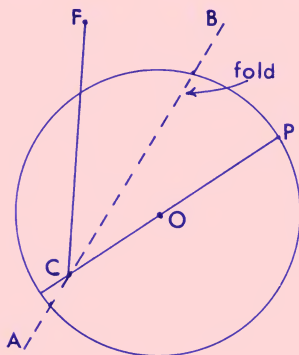
Fig. 9

yond O so the line will intersect AB at C. Connect C with F. Now PC minus CO equals the radius, a constant. But, by the way the curve was formed, CP equals CF. Therefore CF minus CO equals the radius. Thus C is located so that the difference of the distances from F and O is constant (Fig. 9). The figure formed by all such points C is the hyperbola.

CONIC SECTIONS IN LIFE

Experiment 21. The quickest way to make a parabola is to throw a coin or other object into the air. The path of any object moving freely under the influence of gravity is always a perfect parabola. The path becomes distorted from that of a parabola only when other forces such as air resistance are present. Reflectors are often parabolic in shape. If a concentrated source of light is placed at the focus of a parabolic reflector, the reflected rays are parallel. Parabolas, like circles, always have the same shape. When you compare

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the low flat path of a pitched ball with the high curve of a "fly," you might be tempted to doubt this. But the shape that any given parabola seems to have depends on how much of the curve you are using. Parabolas vary only in size, and any given parabola can be obtained by enlarging or reducing another one.

Experiment 22. Notice which conic sections are used in architecture. Elliptical and sometimes circular arches are used over entrances. Bridges are sometimes constructed with elliptical arches, but many are built with parabolic arches to furnish strength as well as grace.

This unit was designed with the cooperation of the National Council of Teachers of Mathematics.
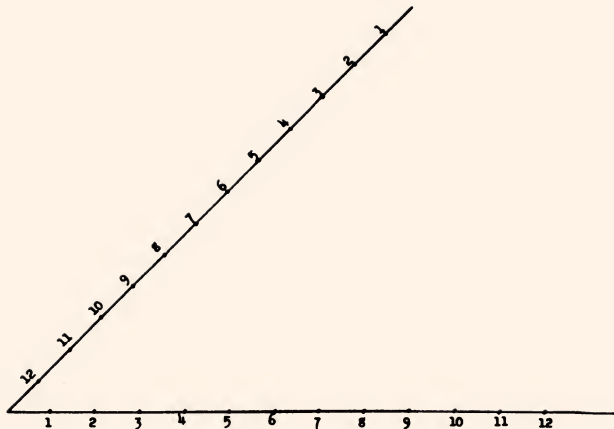
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PARABOLA

HYPERBOLA

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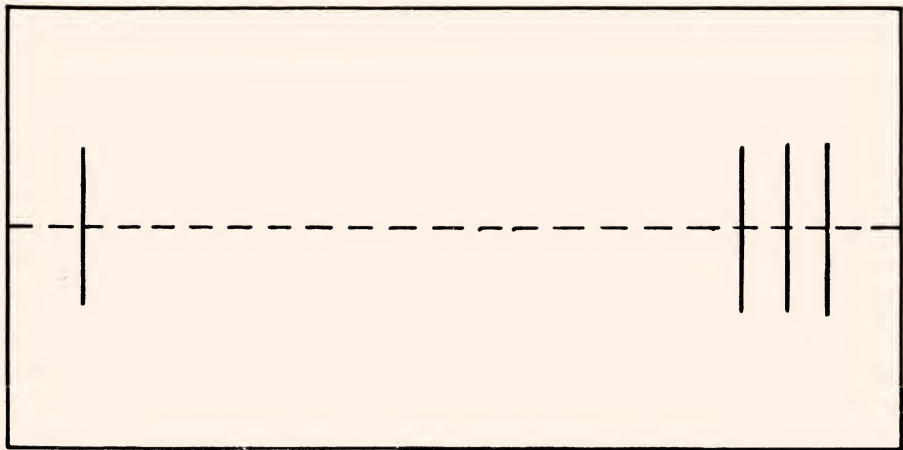
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